# Finite Hex Element Adaptive Mesh Refinement of Demagnetizing Field Computation

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## ABSTRACT

Micromagnetic simulations are an important tool to design better magnets. The computationally most expensive part is the evaluation of the demagnetizing field in and outside of the magnet. Therefore, in this work we propose to reduce computing resources by using the framework of MFEM[1]. This framework allows the use of hexagonal finite elements and an has a built-in Adaptive Mesh Refinement module. We demonstrate the demagnetization field computation convergence towards the exact solution at different mesh refinement stages, where refinement itself is triggered with the Zienkiewicz-Zhu[2] error estimation. This first proof of concept shows a convergence rate of -0.07.

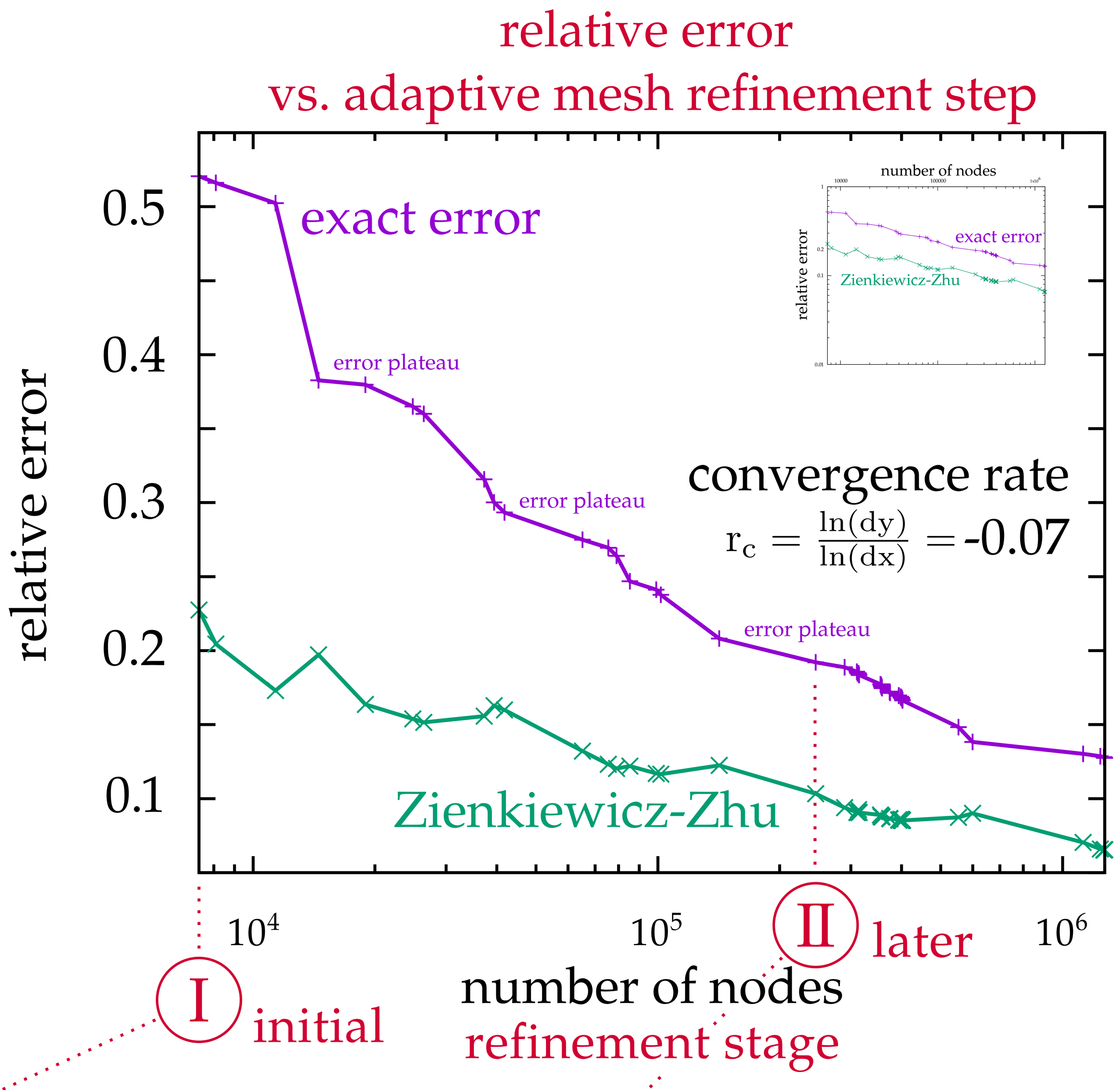
## RESULTS

The relative error for the demagnetizing field computation decreases with increasing number of elements. The adaptive mesh refinement is triggered by Zienkiewicz-Zhu residuum estimation and the mesh is refined mainly around the surfaces of the magnet normal to its initial magnetization.

Relative error will never reach zero, because of singularity of the demagnetizing field near magnet's edges.

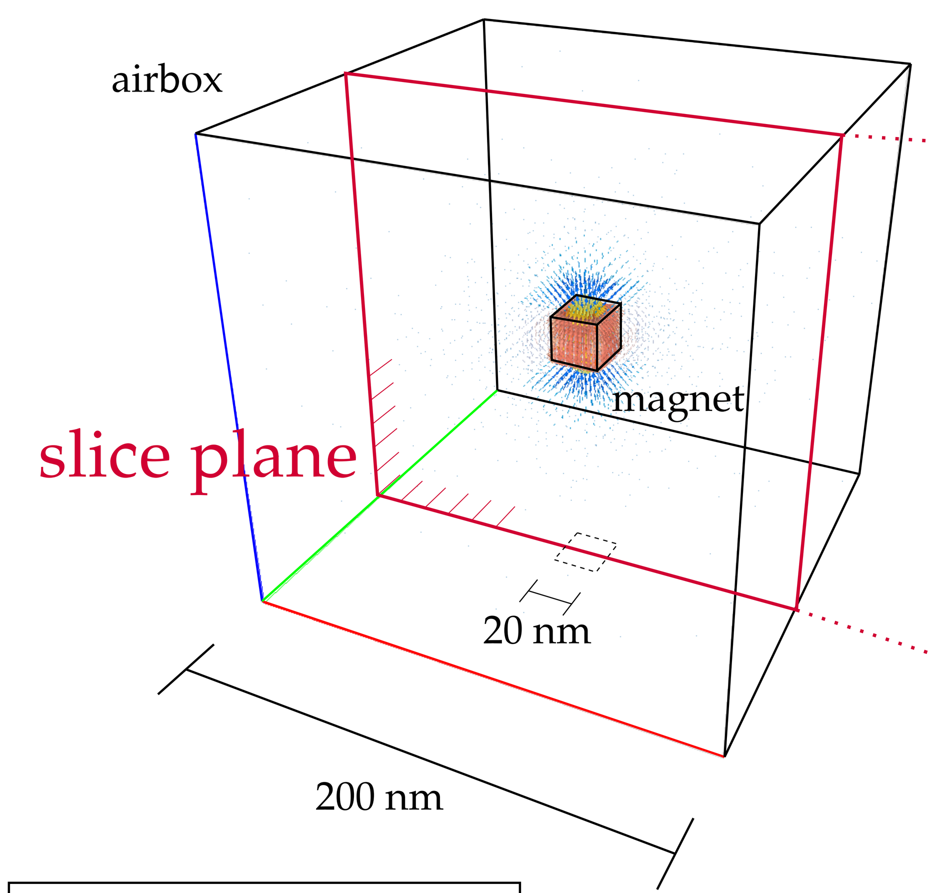
Refinement rule currently not well chosen, see error plateaus on the right.

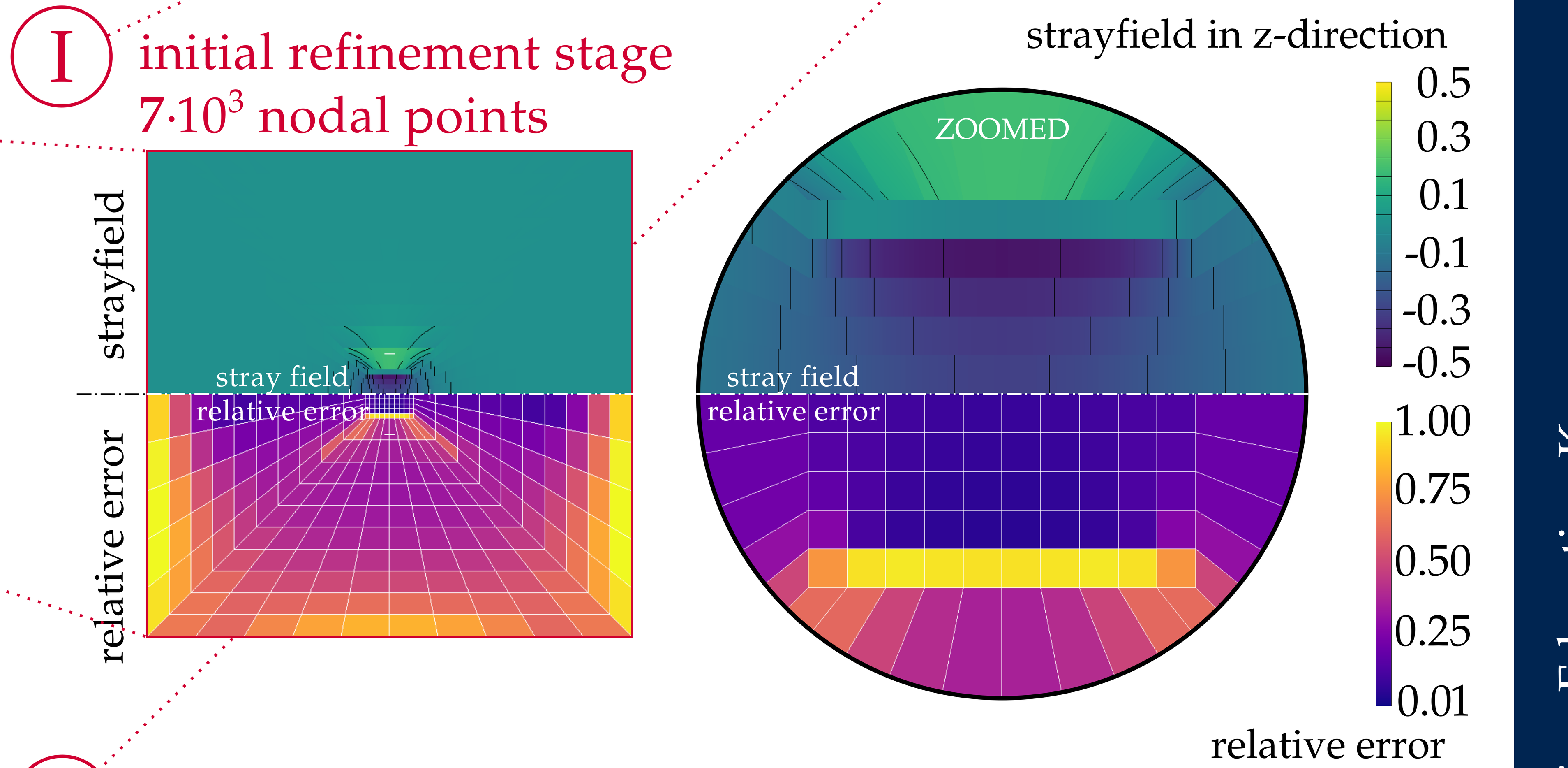
A convergence rate of -0.07 is computed.

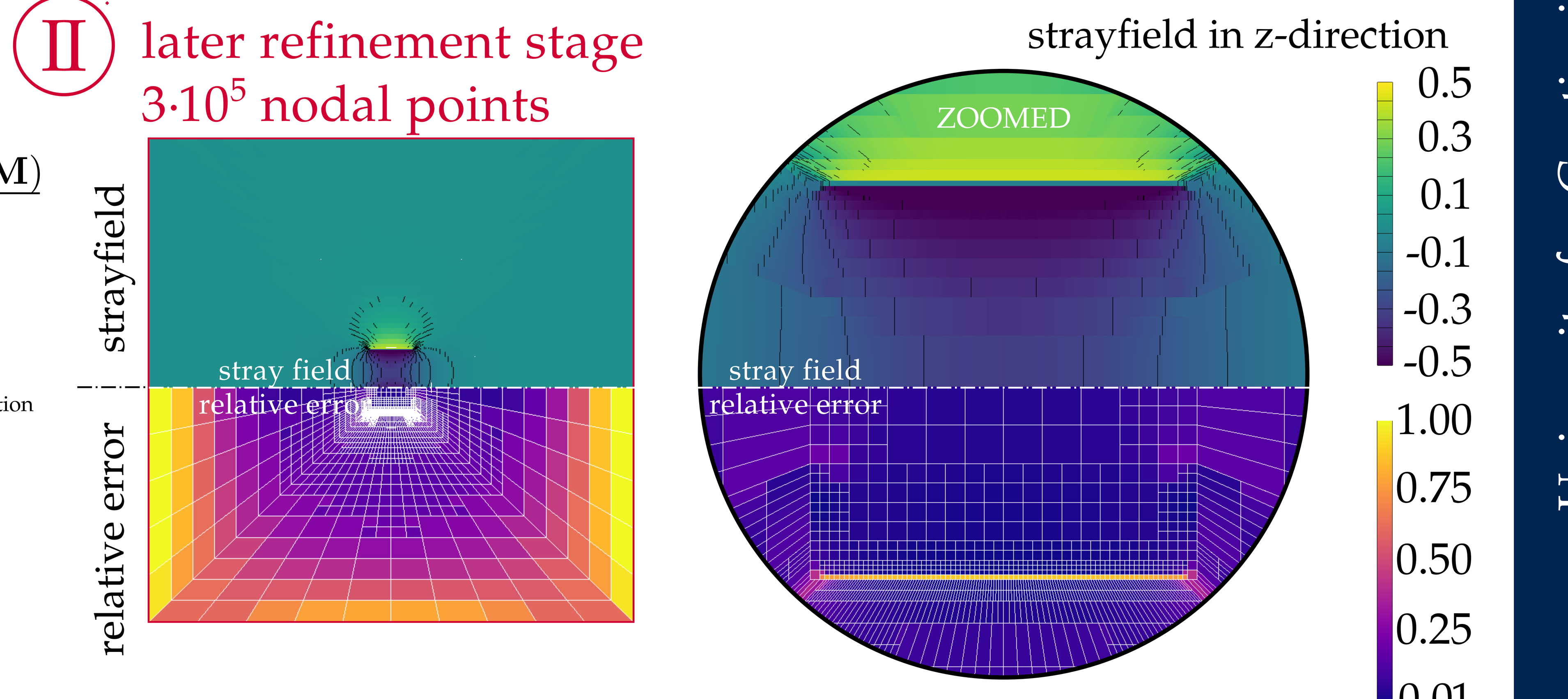



## SETUP

The magnet is surrounded by an air box.
(zero field boundary conditions at infinity)



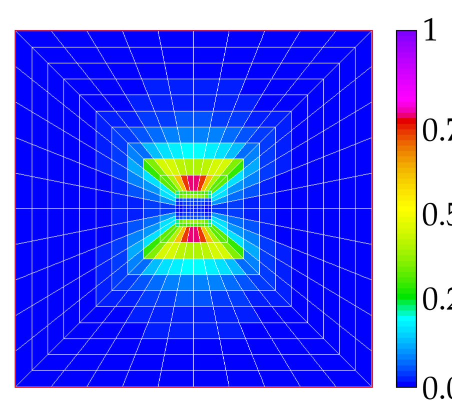




### Zienkiewicz-Zhu error estimation

Computes the error by comparing the smoothed demagnetizing field with the finite element solution at the elements.

Elements are marked for refinement if the element error is greater than 70% of the current maximum error.




Because of the singularity of the demagnetizing field near the edge, the error will never reach zero[4], but the error is reduced by refined elements at the edges.

## EQUATIONS

(1) Scalar potential

$$\mathbf{H}_{\text{demag}} = -\nabla \mathbf{u}$$

(Gauss' law for magnetism)

$$\nabla \cdot \mathbf{B} = 0 \qquad \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

(2)

$$\nabla \cdot \mathbf{H}_{\text{demag}} = -\nabla \cdot \mathbf{M}$$

(3)

(4)

$$\nabla^2 \mathbf{u} = \nabla \cdot \mathbf{M}$$

(Poisson equation) can be solved with Green's function

slice plane, outside, in

"in"-side magnet

$$\nabla^2 \mathbf{u}_{\text{in}} = \nabla \cdot \mathbf{M}$$

"out"-side magnet

$$\nabla^2 \mathbf{u}_{\text{out}} = 0$$

(5) Boundaryconditions ensuring continuity of the ...

(i) tangential component of $\mathbf{H}_{\text{demag}}$ from $\nabla \times \mathbf{H}_{\text{demag}} = 0$

$$\mathbf{u}_{\text{in}} = \mathbf{u}_{\text{out}}$$

Interface Condition at magnet surface

(ii) normal component of $\mathbf{B}$ from $\nabla \cdot \mathbf{B} = 0$

$$(\nabla \mathbf{u}_{\text{out}} - \nabla \mathbf{u}_{\text{in}}) \cdot \mathbf{n} = \mathbf{M} \cdot \mathbf{n}$$

Dirichlet Boundary Condition at "infinity" (air box surface)

(6) FEM: Galerking method applied to transfer the demagnetization problem to a systemt of linear equations.

$$\int_{\Omega_{\text{in}}} dV \varphi_i \nabla^2 \mathbf{u} + \int_{\Omega_{\text{out}}} dV \varphi_i \nabla^2 \mathbf{u} = \int_{\Omega_{\text{in}}} dV \varphi_i \nabla \cdot \mathbf{M} + \int_{\Omega_{\text{out}}} dV \varphi_i \nabla \cdot \mathbf{M}$$

Hat function

(7)

$$\int_\Gamma dS \varphi_i (\nabla \mathbf{u}_{\text{in}} - \nabla \mathbf{u}_{\text{out}}) \cdot \mathbf{n} - \int_{\Omega_{\text{in}} \cup \Omega_{\text{out}}} dV \nabla \varphi_i \nabla \mathbf{u} = \int_\Gamma dS \varphi_i \mathbf{M} \cdot \mathbf{n} - \int_{\Omega_{\text{in}}} dV \nabla \varphi_i \cdot \mathbf{M}$$

(8) scalar potential interpolated by piecewise linear functions over finite elements

$$\mathbf{u}(\mathbf{x}) = \sum_i \varphi_i(\mathbf{x}) \mathbf{u}_i$$

## CONCLUSION

Demagnetization field successfully computed with HEX-Elements using MFEM. Adaptive Mesh Refinement rules tested, but have to be improved. The estimated error and the exact error decrease at a similar rate. In micromagnetics the refinement will stop if mesh size has reached exchange length.

## CREDITS TO

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## ACKnowledgements

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